In 1748, Leonhard Euler published his remarkable *Introductio ad analysin infinitorum* (Introduction to Analysis of the Infinite) [14]. The first sentence of the preface could have been written last week: “Often I have considered the fact that most of the difficulties which block the progress of students trying to learn analysis stem from this: that although they understand little of ordinary algebra, still they attempt this more subtle art.” And so Euler wrote his book, often called the world’s first pre-calculus textbook, in which he clearly set out the topics we still associate with that course—functions, logarithms, exponents, trigonometric functions, polynomials, inverse functions—and paving the way for students to learn about the newly-discovered branch of mathematics called the calculus. The text, still widely read and admired as a model of clear exposition, remains a classic of mathematics.

Another book about the new calculus and the mathematics supporting it appeared that year: Maria Agnesi’s *Instituzioni Analitiche*, the Foundations of Analysis [2]. Published in Milan “for the use of young Italians,” it also was meant to be used as a textbook and aimed to lead its readers from basic, familiar concepts to newer, more advanced ones. Although it caused quite a stir in certain circles, and was actually translated into English before Euler’s text, it is largely forgotten now. If it is remembered at all, it is for an amusing mistranslation—one that hardly describes its author at all.

**The witch**

Maria Gaetana Agnesi was born in 1718, when Leonhard Euler was eleven. Painfully shy and devoutly religious, she wanted only to enter a convent and serve the poor. But her father, widowed three times over and left with twenty-one children, convinced her to stay at home and run his household. And so the obedient young Maria devoted the next twenty years of her life to that task, and to the education of her younger brothers and sisters. She never married. After her siblings grew up and moved away, she established a home for indigent women and children and lived there as its director until her death at age 80.
One of the topics Maria was determined to teach her siblings was the exciting new branch of mathematics that was beginning to become known in Europe: the calculus. As she says in the preface, “There are few so unacquainted with Mathematical Learning, but are sensible the Study of Analyticks is very necessary, especially in our days” [1, p. xxi]. She read everything available to her about the subject—which, admittedly, wasn’t much. She read Newton and Leibniz and, as Euler’s works became available, read them, too—and then wrote a text for her siblings on the foundations of this new kind of mathematics. She set up presses in her father’s house and printed a beautiful, richly illustrated text that presented word problems (the first one asked how to divide riches equitably among the poor) and their solutions, on topics beginning with elementary algebra and solving equations, moving on to plane and solid geometry, and finally reaching, by Chapter VI, “Methods of maxima and minima and tangents to curves.” In Chapter V, “On the construction of loci which exceed the second degree,” she described in Problem III an interesting curve that had been introduced by Fermat [6], then rediscovered by the Italian mathematician Guido Grandi—the versiera, perhaps from the Latin verb *vertere*, to turn [24]. Oddly enough, this curve made Agnesi famous.

In 1748, John Colson, following in the footsteps of Isaac Newton, Isaac Barrow, and others, was the Lucasian Professor of Mathematics at Trinity College in Cambridge. Colson had made his name by translating much of Newton’s work from Latin into English. When he heard that an Italian woman had written about fluxions, he was so inspired that he taught himself Italian in order to translate her book. When Colson got to Problem III of Chapter V, he found the word versiera unfamiliar. Not finding it in his dictionary, he found a similar word and then declared that this curve was called the wife of the devil, or the witch [1, p. 222]. And so that curve is still known as the witch of Agnesi. The index of virtually every calculus textbook today includes a reference to this exotically-named curve—the one thing our students learn about this extraordinary woman from Milan.

**The mistress**

Several hundred miles to the northwest, a young French woman had also discovered the new calculus and wanted to make it more accessible and comprehensible to her contemporaries. It is hard to imagine a woman less like Maria Agnesi than Gabrielle Émilie Le Tonnelier de Breteuil, marquise du Châtelet (1706–1749). Wealthy, aristocratic, and unconventional, she devoted her youth to balls, gambling, card-playing, the theater, fine food, and wine. Married at eighteen to a marquis, she bore him three children. As was the custom in her social set, she took lovers as well, the most famous of whom was the writer and philosopher Voltaire. In her thirties, she abandoned her glamorous Parisian life and turned to the life of the mind.

Prevented by her gender from participating in the intellectual life of Paris, or even attending a university, du Châtelet set up her own academy at her husband’s estate in Cirey, where she and Voltaire entertained scholars as they studied, experimented, and wrote. They both became fascinated by Isaac Newton, his philosophy, and his mathematics. The fact that du Châtelet also read works by Gottfried Leibniz and often supported his point of view became a source of conflict between the lovers. After collaborating with Voltaire on a treatise about the metaphysical elements in Newton’s philosophy, and competing with him by writing her own essay on the nature and propagation of fire for a contest of the French Academy of Science, du Châtelet wrote her encyclopedic text, *Institutions de Physique (Foundations of Physics)* [8] for her young son, because no existing text integrated all of the current knowledge in that subject. She...
then devoted herself to translating Newton’s *Principia Mathematica* from Latin into French, including her own extensive commentary and explanation. Published after her death, it remains the only complete French translation of Newton’s masterpiece [10].

Émilie du Châtelet was arguably one of the most brilliant and accomplished women of her time—perhaps of any time. And yet her biographies typically bear sensational titles like *Passionate Minds* [5], *Voltaire in Love* [21], or *Divine Mistress* [12]. She is featured in a recent book [18] as one of several *Amazing Mistresses*. One can only wish that she were as famous for her mathematical accomplishments as for her romantic ones.

**The forgotten physicist**

In another part of Italy, while Agnesi and du Châtelet were learning about and teaching calculus, another devoted Newtonian was studying—and mastering—mathematics and physics. Laura Bassi (1711–1778) lived in Bologna, home to an extraordinary university. Although she could not study there officially, she worked with private tutors and was allowed to attend some classes. Her work was so outstanding that she was elected a member of Bologna’s Academy of Science at the age of 20. Later that year, the university granted her a lectureship, and in December of 1732 she delivered her first lecture, on Newtonian physics, becoming the first woman to teach officially at a European university, although she did not teach regularly and her salary was very low. We know that she spent three years studying algebra with the head of the department of mathematics, Gabriele Manfredi, and we know that two of the annual public lectures she delivered were her 1752 lecture “Mathematical dissertation” and, in 1757, an “Algebraic dissertation.” Unfortunately, almost all of her texts have been lost. The two extant ones, on hydrometrics and mechanics, are very mathematical [13].

Laura Bassi was a superstar during her lifetime, well-known in scientific circles all over Europe. Voltaire wrote to her as follows, asking for her intervention in procuring an appointment to the Bologna Academy of Science, “Most Honored Lady: I would like to visit Bologna so that I might say to my fellow citizens that I have seen Signora Bassi, but, deprived of this honor, I trust that I may with justice cast at your feet this philosophical homage in reverence to the glory of her century and sex…” [26]. She was eventually appointed chair of the Department of Experimental Physics at the Institute of Sciences at the university. A street and a school were named for her. But she has almost become lost to us. It is hard to find anyone who recognizes her name today.

These three women were separated—by geography, language, and culture—yet they were undoubtedly aware of each other. When Maria Agnesi published her *Instituzioni*, she received a letter of congratulations and appreciation from Laura Bassi, noting that she had known of Agnesi’s talent “for many years” [20, p. 120]. After the publication of Émilie du Châtelet’s *Institutions* and its translation into Italian, she was elected to the Bologna Academy of Science. Her biographer Judith Zinsser writes, “Perhaps because she knew that the Newtonian Laura Bassi was also a member, and a lecturer at the University of Bologna, Du Châtelet noted what an ‘encouragement’ this was for ‘persons of my sex’ … ‘to engage in and cultivate the Sciences from which prejudice had up to the present appeared to exclude them.’” [28, p. 210]. She also notes that Bassi used the *Institutions* in her classes at the University.

These women were aware, of course, of the great Leonhard Euler. Agnesi acknowledged Euler’s work on infinite quantities in her textbook [20]; in several of du Châtelet’s letters, she mentions having read in the newspaper that Euler was in Berlin, and that she wanted to arrange to send him a copy of her *Institutions* [9]. She
cited his work on the ebb and flow of tides in her commentary on the \textit{Principia} \cite{11}. There is even some evidence that Euler was aware of, and corresponded with, at least one of these celebrated women. The Euler Archive notes that “the Euler–du Châtelet correspondence consists of 3 letters, of which 2 were written by Euler” between 1740 and 1744 \cite{17}. The eighteenth-century world of European mathematics was a small one, and those writing about analysis could hardly avoid each other.

\textbf{The princess}

When Euler lived in Berlin, he sometimes spent evenings playing music with his friend the future Margrave Friedrich Heinrich von Brandenburg-Schwedt, a cousin of Frederick the Great. The Margrave had a teenage daughter who wanted to learn more about science and mathematics but, like the three women we have already met, the young princess (variously called Charlotte Ludovica Luisa or Friederike Charlotte Leopoldine Luise, 1745–1808) was excluded from schools that taught those subjects. When her father asked Euler if he would tutor his daughter in those subjects, he could hardly refuse \cite{7}. But, as Euler writes, “The hope of having the honour to communicate in person to your highness my lessons in geometry becoming more and more distant, which is a very sensible mortification to me, I feel myself impelled to supply personal instruction by writing, as far as the nature of the subjects will permit” \cite[Letter 1]{15}. Thus began a correspondence of almost 250 letters over the course of two years (1760–62) on topics in mechanics, astronomy, physics, optics, and acoustics, and which have come down to us as \textit{Euler’s Letters to a German Princess} \cite{15}. They contain the first instance of Euler diagrams, similar to Venn diagrams, illustrating logical statements. They became wildly popular as textbooks and were almost immediately translated into many languages, including Russian, Italian, Dutch, and Spanish.

The young princess probably had little time to reflect on the new science and mathematics contained in those letters. At the tender age of nineteen, she was named the last abbess of the great Lutheran abbey at Herford, “whose Canonesses must be all Princesses, or Countesses of the Empire” \cite[p. 27]{23}. Perhaps unwittingly, she followed in the footsteps of another notable Abbess of Herford who had received letters from a famous mathematician. In the previous century, Elizabeth of the Palatinate (1667–1680) became known for her mathematical precocity and her correspondence with Descartes on philosophy and mathematics. In fact, Descartes dedicated his 1644 \textit{Principia Philosophiae} to her \cite{22}. Princess Charlotte served in her role as abbess until 1802, when the abbey was secularized and she was forced to flee. She died several years later.

\textbf{Math for women!}

The first English translation of Euler’s letters, by The Rev. Henry Hunter, was published in London in 1802. Hunter writes \cite[p. xvii]{15}:

\begin{quote}
Euler wrote these Letters for the instruction of a young and sensible female, and in the same view that they were written, they are translated, namely, the improvement of the female mind; an object of what importance to the world!
\end{quote}

While it may or may not have been Euler’s intention to write a text especially for females, one person who definitely wrote with that aim was the Italian Francesco Algarotti (1712–1764). Algarotti studied mathematics and natural sciences at the Uni-
versity of Bologna, particularly Newtonian optics. He replicated some of Newton’s experiments on the refraction of light. He later moved to Paris, where he became the darling of the intellectual set and was befriended by du Châtelet and Voltaire. He visited them at Cirey, where he spent several weeks discussing science in general and optics in particular. In 1737 he published his masterpiece Neutoniansimo per le dame (Newtonianism for the Ladies) [3].

This runaway best-seller consisted of a set of dialogues between a young cavaliere and the marchesa di E** [19] on Newtonian optics. Émilie du Châtelet probably had reason to believe that these two characters were based on Algarotti and herself, so she was understandably surprised and indignant when the young man in the book was portrayed as a brilliant expositor, explaining science to a rather dim-witted woman. She also expected that the book would be dedicated to her [27]. Instead, it was dedicated to the “Bolognese Filosofessa,” Laura Bassi. It even contained a poem praising Bassi [3, Dialogo Primo]. The book was popular not only in Italy, where ladies read it and then adopted English practices like drinking tea and wearing English hats, but also in England itself, where it was translated in 1739 by Elizabeth Carter as Sir Isaac Newton’s Philosophy explain’d for the use of Ladies in Six Dialogues on Light and Colours [4].

For a final example of mathematics written exclusively for women in this period, we return to the Lucasian Professor John Colson. Once Colson had finished translating Agnesi’s Instituzioni Analitiche, he decided it was still too difficult for English ladies to understand and appreciate as he wanted them to. He wrote [1, p. ii]:

'I confess I also entertained some distant hopes, that it might excite the curiosity of some of our English Ladies; that it might raise an emulation in them, a laudable ambition to promote the glory of their country, with a generous resolution not to be outdone by any foreign ladies whatever.'

And so he decided to write a commentary on Agnesi’s work:

'Now that these [women], and all other readers, may attain these advantages with as little trouble as possible, I shall endeavour to draw out the Plan of this Work at full length, and in a popular manner, inserting some useful Observations to explain the Art itself; so that the Work, when published, may be easily read and apprehended, by such as will peruse it with the necessary diligence and attention [1, p. iii].

Toward the end of his life, Colson began work on The Plan of the Lady’s System of Analyticks (published in [1], pp. i–xlvi). Painstakingly he went through Agnesi’s book, paragraph by paragraph, explaining it in his own words. It was not especially helpful or enlightening, and so it is not a great disappointment that he died before completing this task.

**Women for mathematics!**

History has not been kind to the women most closely associated with mathematics in the time of Euler: One is remembered as a witch, one as an adulteress, one hardly at all. All had to overcome enormous obstacles to learn mathematics. Euler himself was pressed into service when it came to teaching girls math and science. Their works still endure, however, as a testament to the beauty and power of mathematics—and its pull on those who are determined to master it.
Summary. We explore mathematics written both by and for women in eighteenth-century Europe, and some of the interesting personalities involved: Maria Agnesi, Emilie du Châtelet, Laura Bassi, Princess Charlotte Ludovica Luisa, John Colson, Francesco Algarotti, and Leonhard Euler himself.

References


Gender Bias?

Suppose you are teaching second-semester calculus. A third of the class are women and, as the semester progresses, you notice that they comprise half of the best students and none of the poorest students. Superficially it seems there is no gender bias in mathematics at your school! But what about selection bias? Your sample includes only students successful enough in prior courses to enroll in second semester calculus. Shouldn’t women be proportionally represented throughout the grade distribution (as men are)? The lack of women in the mid-to-low performance range might indicate that they are somehow screened out of second semester calculus, while men of similar performance levels aren’t.

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